Simulaciones cuánticas robustas (y frágiles) en presencia de errores

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Charla @ QUFIBA
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THE UNIVERSITY OF NEW MEXICO

QUANTUM SYSTEMS ACCELERATOR
Catalyzing the Quantum Ecosystem

NSF
PHY-1630114

Background picture: **Ghost Ranch, New Mexico (USA)**

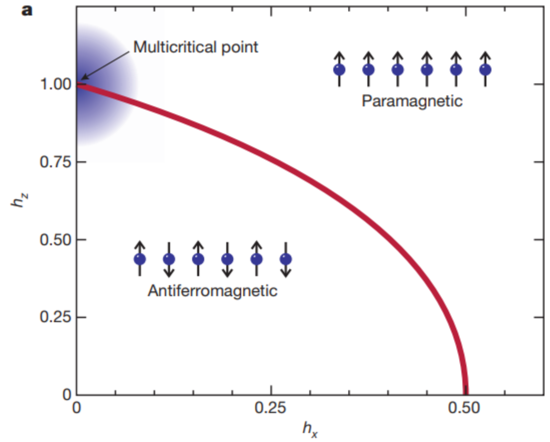
QUANTUM SIMULATION

Quantum simulators: study models of many-body quantum systems by *engineering* a physical system that is governed by the same laws, and that can be manipulated in the lab

R. Feynamn, Int. J. Theor. Phys. 1982

Resource scaling → Time / memory ~ $O(exp(N))$ (Classical) vs $O(poly(N))$ (Quantum)

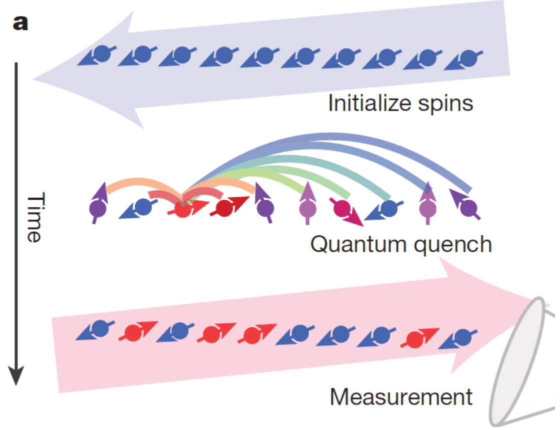
Equilibrium properties



J. Simon et al, Nature 2011

Thermal behavior, quantum phase transitions (QPTs), …

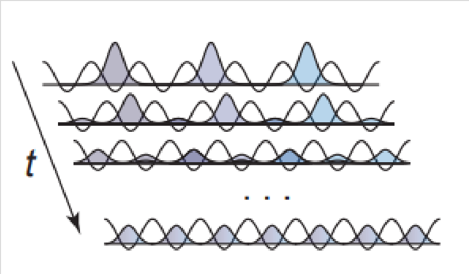
Dynamical properties



J. Zhang et al, Nature 2017

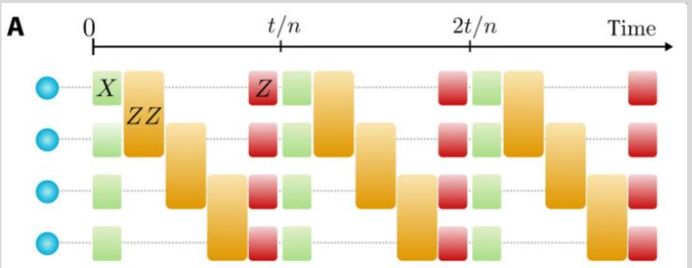
Transport, thermalization / equilibration, dynamical QPTs, …

Analog simulators



S. Trotzky et al, Nat. Phys. 2012

Gate-based simulation ("digital")



M. Heyl et al, Sci. Adv. 2019

NISQ ERA QUANTUM TECHNOLOGIES

Decoherence

Washes out quantum properties (coherence, entanglement), lost of quantum advantage

Sources: **noise**, **inhomogeneity** or unwanted **coupling** to other systems **(environment)**

Quantum error correction (QEC): as long as the noise is *small enough*, one can use some extra quantum resources to preserve quantum information

QEC allows for fault tolerant quantum computing (in particular, universal quantum simulation)

Overheads are polynomial but still seem (?) way into the future ($\sim 10^5 - 10^6$ physical qubits with $10^{-3} - 10^{-4}$ error rates)

Current era of **'noisy intermediate-scale quantum' (NISQ)** devices

J. Preskill, Quantum 2018

→ 10-100s of qubits: large Hilbert space but too small to do fault-tolerant QC

→ **Errors** – restricted depth / evolution time. How do errors accumulate in 'analog' systems?

Analog errors in quantum annealing: Lidar, Albash, Hen, others
npj Quantum Info. 2019, Quantum Sci. Tech. 2019

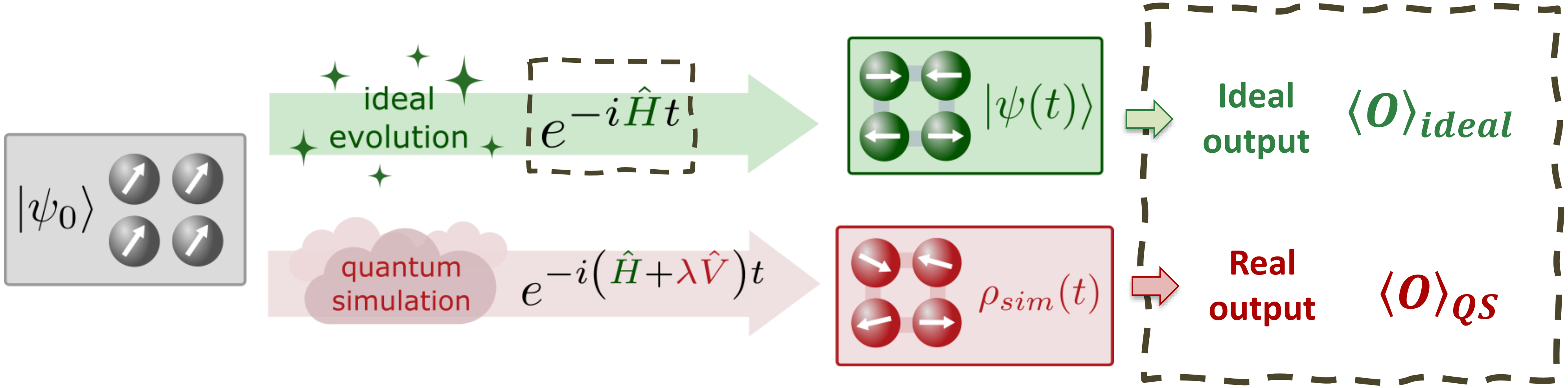
Contenders

- Hybrid quantum – classical algorithms (variational algorithms, VQE, QAOA) → S. Endo et al, Hybrid quantum-classical algorithms and quantum error mitigation, arxiv 2011.01382
- Adiabatic quantum computing (or variations)
- Quantum simulation!

QUANTUM SIMULATION IN THE NISQ ERA

Absence of error correction means that some things will be necessarily off in the simulated state

"Can one trust quantum simulators?" Hauke et al, Rep. Prog. Phys. (2012)



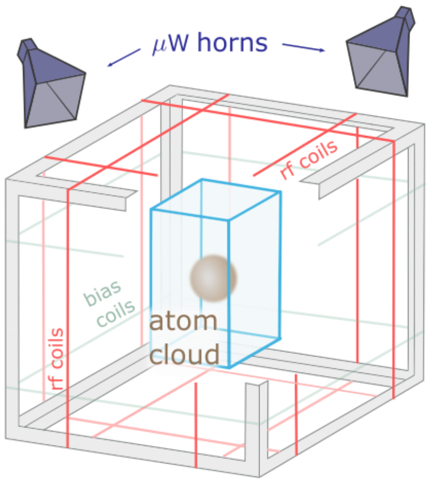
How do **errors** affect different quantum simulation **outputs**?

Which **observables** are **robust** and which are **fragile**?

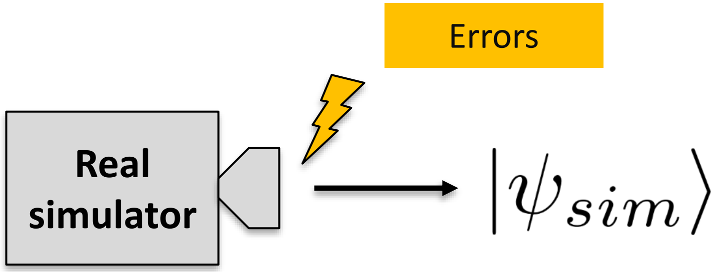
Does the **system complexity** influence the **reliability** the simulation?

OUTLINE

1. Quantum simulation based on optimal control of atomic spins – How does the experimental platform works, and how it informed us what to study from theory side



2. Theory of robust and fragile observables – A theory to predict a priori which outputs of a simulator might be more robust than others in a generic scenario

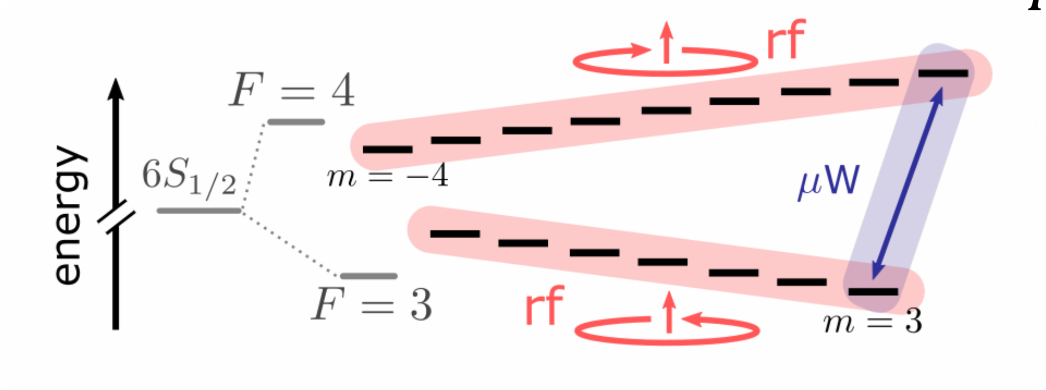


3. Errors and dynamical complexity – An example of how the complexity of the simulated system might 'conspire' against robustness

SMALL-SCALE QUANTUM SIMULATOR

Quantum *hardware*: individual laser-cooled Cs atoms in $6S_{1/2}$ electronic ground state ($d = 16$)

Experiment: P. Jessen group — THE UNIVERSITY OF ARIZONA



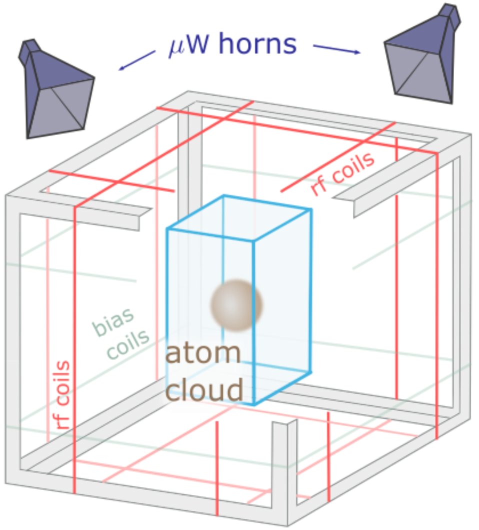
$$H = A\,\vec{I}.\vec{S} + 2\mu_B\,\vec{B}(t).\vec{S}$$

$$\vec{B}(t) \rightarrow B_0\hat{z} + \vec{B}_{rf}(t) + \vec{B}_{\mu W}(t)$$

$$B_{rf}^{x,y}(t) = \Omega_{x,y}\cos(\omega_{rf}t + \phi_{x,y}(t)) \qquad B_{\mu W}(t) = \Omega_{\mu W}\cos(\omega_{\mu W}t + \phi_{\mu W}(t))$$

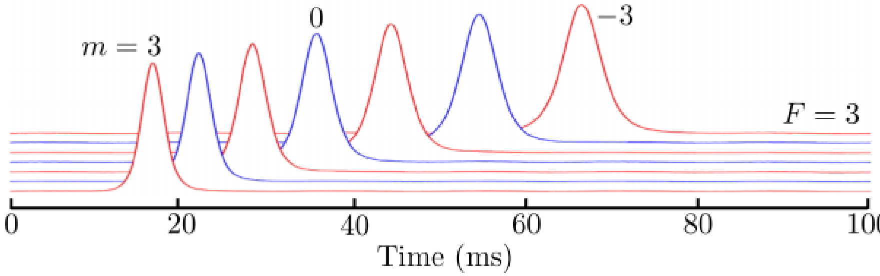
System is 'fully controllable' ↔ for any $W \in SU(16)$, there exists a time T and a set of driving fields $\vec{B}(t)$ such that

$$U(T) = \mathcal{T}e^{-i\int_0^T H(t')dt'} = W$$



Arbitrary state preparation and measurement

1. Optical pumping to $|\chi_0\rangle = |F = 3, m = 3\rangle$
2. Apply fields (found numerically / optimal control) to implement transformation W
3. Measure in $|F, m\rangle$ basis via Stern Gerlach

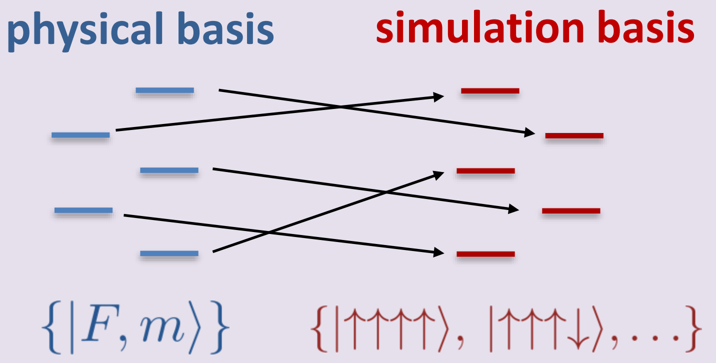


S. T. Merkel *et al*, PRA 2008
A. Smith *et al*, PRL 2013
B. E. Anderson *et al*, PRL 2015
H. Sosa-Martínez *et al*, PRL 2017

QUANTUM SIMULATION VIA OPTIMAL CONTROL

Simulated system → anything that 'lives' in an equivalent Hilbert space: $d = \dim(\mathcal{H}) \leq 16$

- Up to $N = 4$ qubits ($d = 2^N$)
- Up to $N = 15$ qubits in a symmetric subspace ($d = N + 1$)
- (…)



Simulation Hamiltonian H_{sim}

- $H_{sim} = \sum J_{ij}\sigma_i^z\sigma_j^z + \sum h_i\sigma_i^x$
- $H_{sim} = \Gamma\, S_z^p + B\, S_x$
- (…)

Target unitary: $W \equiv e^{-iH_{sim}\delta t}$

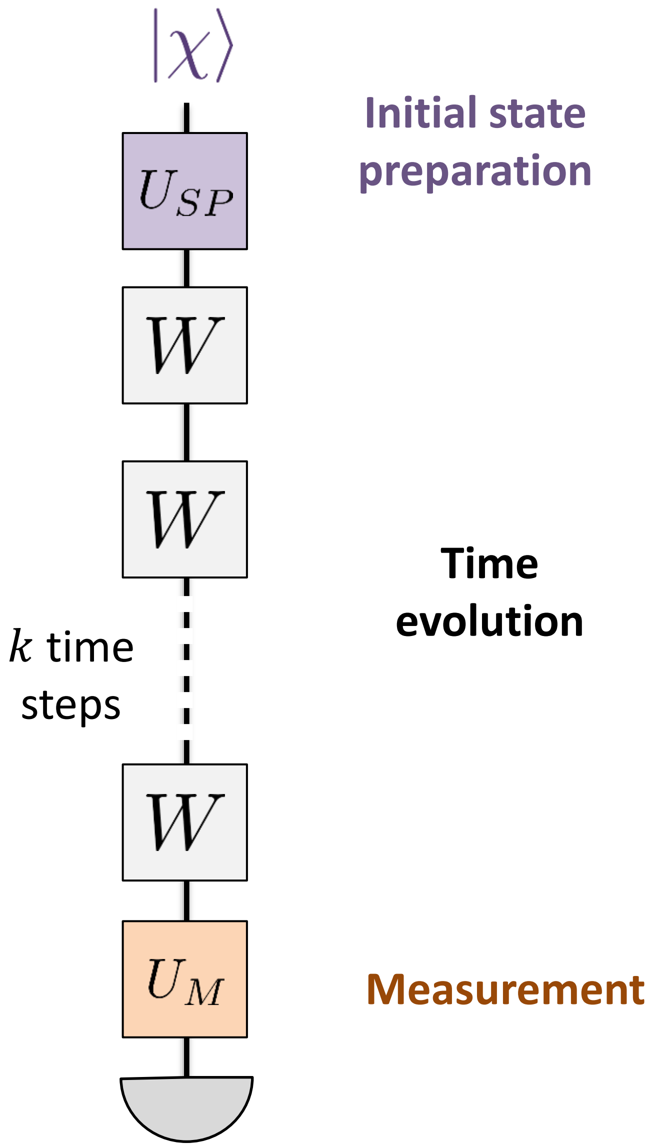
↓

Control fields: $B_x(t), B_y(t), B_{\mu W}(t)$

Repeat control sequence k times

Simulation output

- Map $\{|F, m\rangle\}$ to measurement basis $\{|\phi_\alpha\rangle\}$ via $U_M = \sum |(F, m)_\alpha\rangle\langle\phi_\alpha|$
- Stern-Gerlach measurement gives populations $p_\alpha = Tr(\rho\, |\phi_\alpha\rangle\langle\phi_\alpha|)$
- Outputs (expectation values, etc) are constructed from the populations



QUANTUM SIMULATION PERFORMANCE

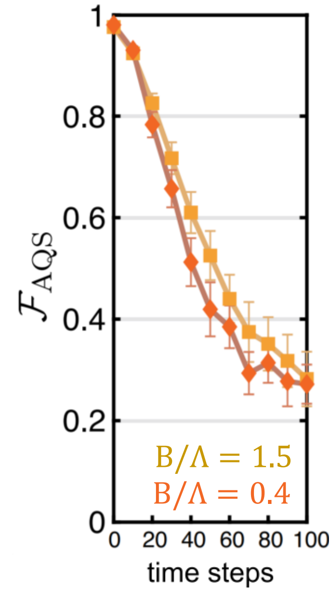
A small, highly accurate quantum simulator

N. Lysne, K. Kuper, PMP, I. Deutsch, P. Jessen, PRL **124** 230501 (2020)

- Universal simulator, programmable through optimal control
- Modest (but non trivial) Hilbert space dimension, <u>not scalable</u>
- Improved optimal control techniques leads to fidelity per time step > 99% and allows ~ 100 time steps (large depth)

Purpose: test **new models**, and explore how **errors** affect the output of quantum simulations in a regime of high accuracy

Example: $H_{\text{sim}} = -\frac{B}{2}\sum_i \sigma_z^{(i)} - \frac{\Lambda}{4N}\sum_{i,j}\sigma_x^{(i)}\sigma_x^{(j)}$ $\quad (N = 15, |\psi_0\rangle = |\downarrow_x\rangle^{\otimes N})$



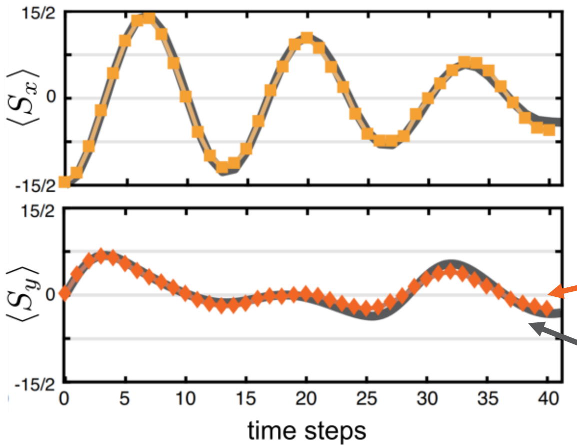
Fidelity

$\mathcal{F}_{\text{AQS}} = |\langle\psi(t_k)|\psi_{\text{sim}}(t_k)\rangle|^2$

ideal → simulated (exp)

- Afer 40 steps, fidelity ~ 60%

Error in the full state is 'high'



Expectation values

$S_\alpha = \frac{1}{2}\sum_i \sigma_\alpha^{(i)}$

simulated (exp)

ideal

Error in **some expectation values** is 'small'

Construct a framework to assess which outputs (**expectation values**) are **robust** and which are **fragile**

ERROR IN THE OUTPUT OF AQS

Ideal simulator → $|\psi\rangle$

Output

$$\langle A\rangle = \langle\psi|A|\psi\rangle$$

(or, a series of $\{\langle A_k\rangle\}$)

Imperfections / perturbations

Real simulator → $|\psi_{sim}\rangle$

close to $|\psi\rangle$ in some metric

$$\langle A\rangle_{sim} = \langle\psi_{sim}|A|\psi_{sim}\rangle$$

Simulator error

$$\delta(A) = \langle\psi|A|\psi\rangle - \langle\psi_{sim}|A|\psi_{sim}\rangle$$

We want to characterize the error as a function of the *output observable A*

AVERAGE SIMULATOR ERROR

Simulator error $\delta(A) = \langle\psi|A|\psi\rangle - \langle\psi_{sim}|A|\psi_{sim}\rangle$

$|\psi_{sim}\rangle = \mathcal{N}(\gamma)\left(|\psi\rangle + \gamma|\psi_\perp\rangle\right)$

Dependence with A, *on average?*

Average over Haar random states
$\overline{X} \equiv \int d\psi X(\psi)$

$$\overline{\delta(A)^2} = \frac{2\gamma^2\mathcal{N}(\gamma)^2}{d^2-1}\left(\mathrm{Tr}\left(A^2\right) - \frac{1}{d}\mathrm{Tr}\left(A\right)^2\right)$$

d = Hilbert space dimension

To compare between different A's

- **Shift** spectrum of A such that minimum eigenvalue is 0 * (leaves $\delta(A)$ invariant)
- Define the **operator** $\rho_A = \frac{A}{\mathrm{Tr}(A)}$

$\rho_A^\dagger = \rho_A, \ \mathrm{Tr}(\rho_A) = 1, \ \rho_A > 0$

* (except if $A = I$)

Average relative error

$$\delta_{rel}(A)^2 = \frac{\overline{\delta(A)^2}}{\overline{\langle A\rangle}^2}$$

$$\delta_{rel}(A) = \sqrt{2\frac{d^2}{d^2-1}\left(\frac{\gamma^2}{\gamma^2+1}\right)\left(\mathrm{Tr}(\rho_A^2) - \frac{1}{d}\right)}$$

"observable purity"

'PURITY' OF AN OBSERVABLE

Average relative error

$$\delta_{rel}(A)^2 = \frac{\overline{\delta(A)^2}}{\overline{\langle A\rangle}^2} \Rightarrow \delta_{rel}(A) = \sqrt{2\frac{d^2}{d^2-1}\left(\frac{\gamma^2}{\gamma^2+1}\right)\left(\mathrm{Tr}(\rho_A^2) - \frac{1}{d}\right)} \qquad \rho_A = \frac{A}{\mathrm{Tr}(A)}$$

$\mathrm{Tr}(\rho_A^2) = \frac{1}{d}$

when $A = \mathbb{I}$ (trivial)

However, $S_x = \frac{1}{2}\sum_{i=1}^{N}\sigma_x^{(i)}$

$\mathrm{Tr}\left(\rho_{S_x}^2\right) = \frac{N+1}{N}\frac{1}{d}$ **(!)**

insensitive → **sensitive**

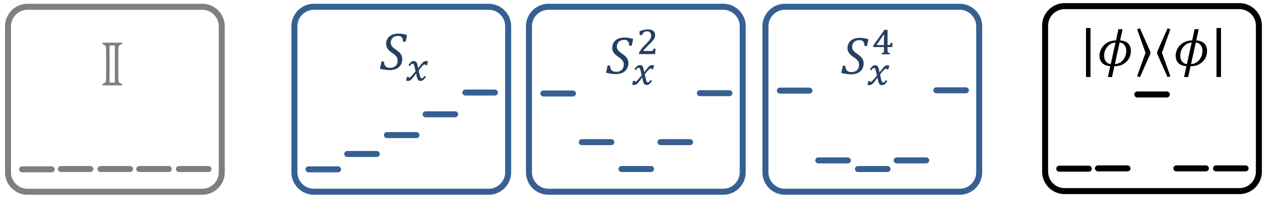
$\frac{1}{d}$ **Purity of A** ⟶ $\mathrm{Tr}(\rho_A^2)$ 1

~ high rank ~ low rank

$\mathrm{Tr}(\rho_A^2) = 1$

when $A = |\phi\rangle\langle\phi|$ is a **projector**

$\langle A\rangle = |\langle\phi|\psi\rangle|^2$



Higher order moments of the distribution become more sensitive

DYNAMICS OF ERRORS

Initial state $|\psi_0\rangle \rightarrow$ **Real simulator** $H(\alpha, \beta, \ldots)$ $\longrightarrow$ $|\psi_{sim}(t)\rangle = e^{-i(\hat{H}+\lambda\hat{V})t}|\psi_0\rangle$

Perturbation → $\lambda\hat{V}$

Simulator error:

$$\delta(A,t) = \langle\psi(t)|A|\psi(t)\rangle - \langle\psi_{sim}(t)|A|\psi_{sim}(t)\rangle$$

$$\mathcal{E}(A,t)^2 = \frac{1}{t}\int_0^t \delta(A,t')^2 dt'$$

Time-averaged cumulative error

Weak random perturbation

- $\langle u_n|V|u_n\rangle$ random, uncorrelated, where $H = \sum_n E_n|u_n\rangle\langle u_n|$
- Leading order perturbation theory

Main result

$$\mathcal{E}_{rel}(A,\infty) = \sqrt{\frac{d}{d+1}\left(\mathrm{Tr}(\rho_A^2) - \mathrm{Tr}(\rho_{A_D}^2)\right)}$$

$$A_D = \sum_n A_{nn}|u_n\rangle\langle u_n|$$

typically $\mathrm{Tr}(\rho_{A_D}^2) \sim \frac{1}{d}$

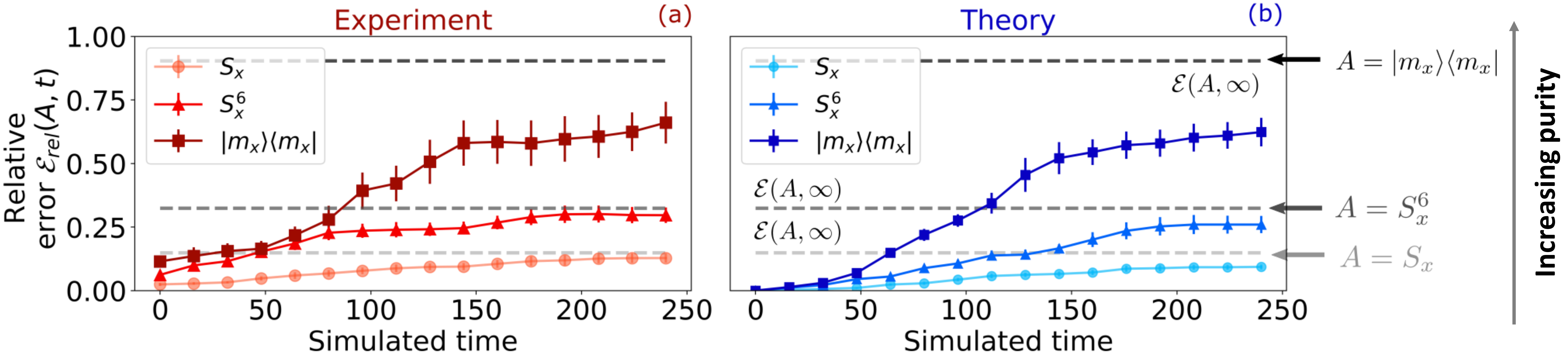
(Average, relative) asymptotic error given by **observable purity!**

Full theory in **arxiv: 2007.01901**

EXPLORING ROLE OF OBSERVABLE PURITY

- Track simulation errors for observables of different **'purity'**: S_x, S_x^{2k}, $|m_x\rangle\langle m_x|$
- Compute cumulative / RMS error $\mathcal{E}(A,t)^2 = \frac{1}{t}\int_0^t \delta(A,t')^2 dt'$
- Average over 10 random states

One fitting parameter → perturbation strength λ



- Dashed lines are analytical predictions
- At long times, **experiment and theory agree well.** At short times, SPAM errors dominate.
- No info about physical errors in the device – **generic model!**

Higher purity → higher errors!

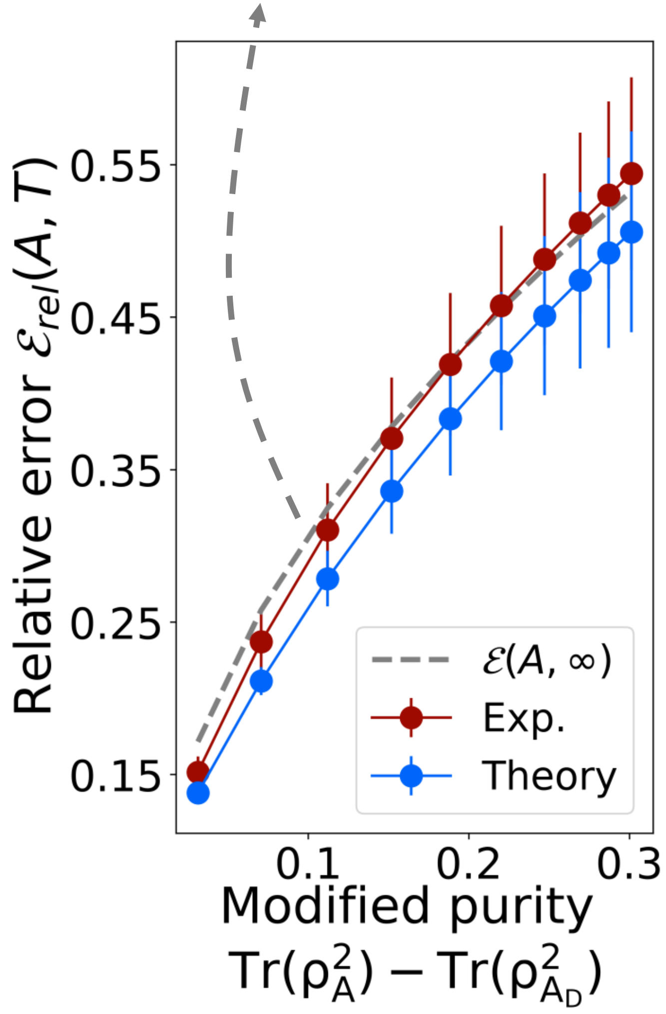
EXPERIMENTAL RESULTS

Higher purity → higher errors!

- We consider operators of the form $A = S_x^{2k}$, purity increases with k
- We measure $\langle S_x^{2k} \rangle$ as a function of time for random initial states, and obtain the long time relative error $\mathcal{E}_{\mathrm{rel}}(A, T)$

Errors obtained from real-world device are seen to be a monotic function of observable purity

$$\mathcal{E}_{rel}(A, \infty) = \sqrt{\frac{d}{d+1}\left(\mathrm{Tr}(\rho_A^2) - \mathrm{Tr}(\rho_{A_D}^2)\right)}$$



Summary of this work

Expectation values of different observables have **different degrees of sensitivity** to imperfections in the state

High purity observables are more sensitive than **low purity** ones (on average)

Predicted behavior is **generic** and is found in **real world devices** – without assuming any model for imperfections

Open questions and future work

- Beyond perturbative regime → effect of *local* perturbations and relation sensitivity vs entanglement
- Random matrix models for perturbations:
  Nation and Porras NJP **20** 103003 2018 / Dabelow and Reimann PRL **124** 120602 (2020)
- Sensitivity of correlation functions: $C(A,B) \Rightarrow \langle AB\rangle - \langle A\rangle\langle B\rangle$ → robustness of mean field vs correlation

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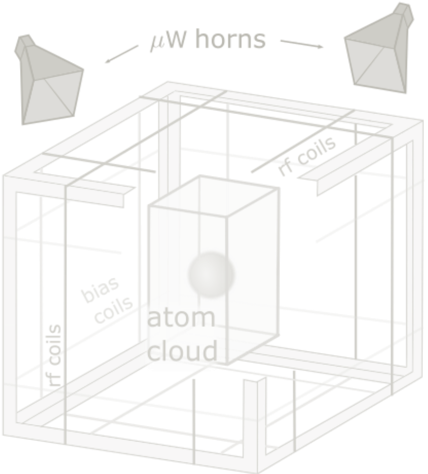

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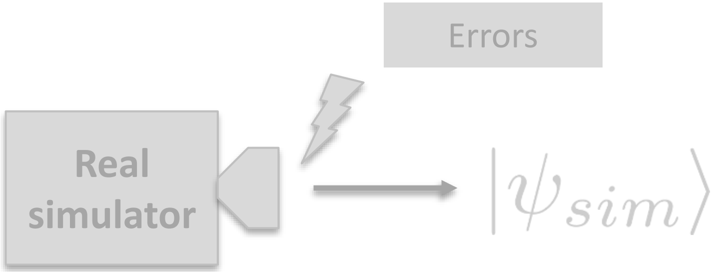

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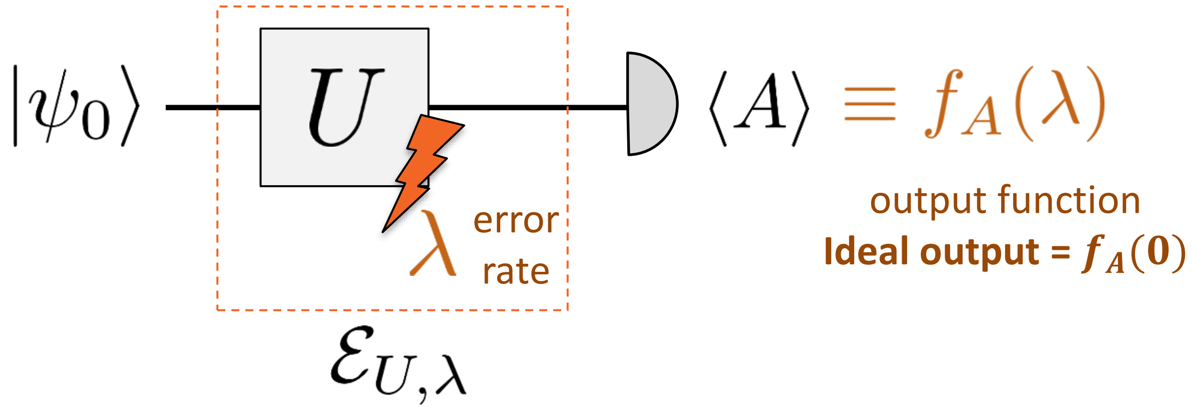
QUANTUM ERROR MITIGATION

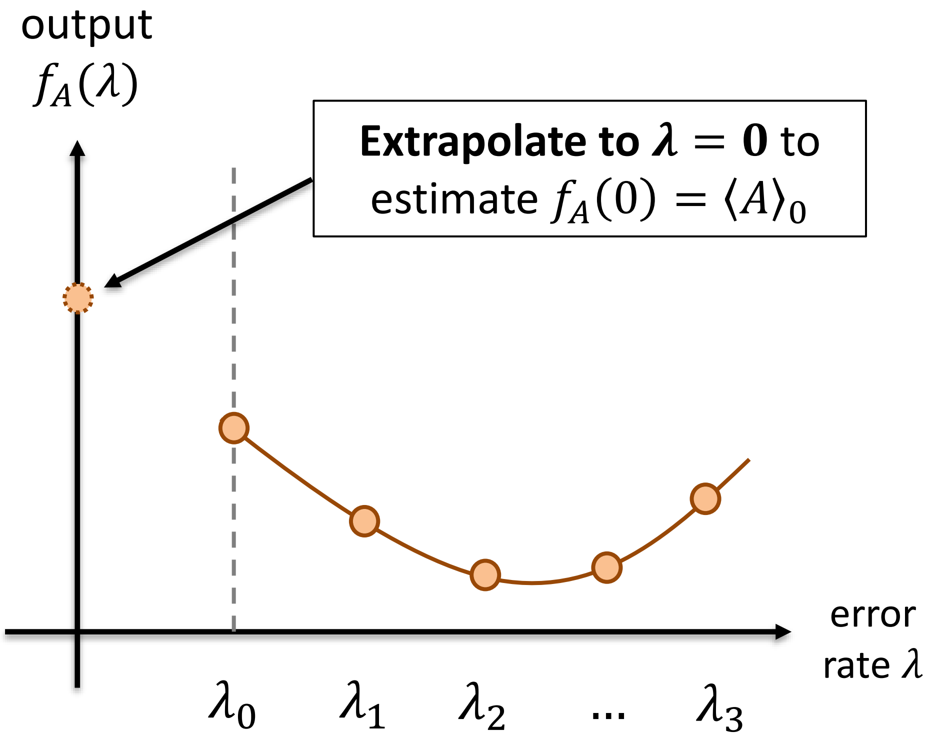
Quantum error correction – quantum resources to correct errors and recover exact state

Quantum error mitigation – classical resources (e.g. post-processing) to reduce errors in the output of a simulation

- **Quasi-probability methods (Temme PRL 2017, Endo PRX 2018)**
- **Learning-based methods (Strikis 2020, Cznarnik 2020) –** e.g. train a neural network to correct output using using classically simulable (Clifford) circuits

- **Zero-noise extrapolation (Temme PRL 2017, Li PRX 2017)**





We can't decrease λ, but in some cases we can increase it artificially (identity insertion, Hamiltonian rescaling, …)

Run simulations for different noise levels $\{\lambda_i \geq \lambda_0\}$ and obtain different *samples* of the output function $\{f(\lambda_i)\}$

COMPLEXITY AND ERROR MITIGATION

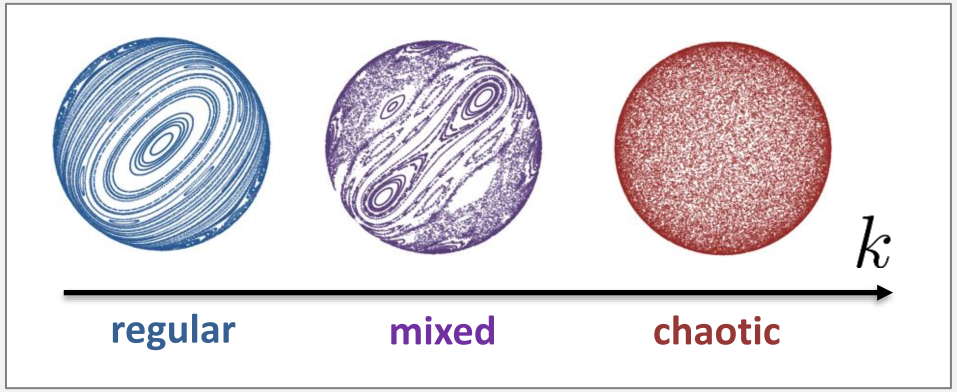
Zero-noise extrapolation is expected to work well for completely Markovian errors, and it has been implemented successfully in small systems

In a generic setting, one expects this to work **as long as errors are small enough** such that some **perturbative expansion is valid** – is there a relation between this and the complexity of the system?

Breakdown of perturbative expansions are a signature of quantum **chaotic** systems (i.e., valid up to a perturbation strength that scales inversely with Hilbert space dimension)

Example

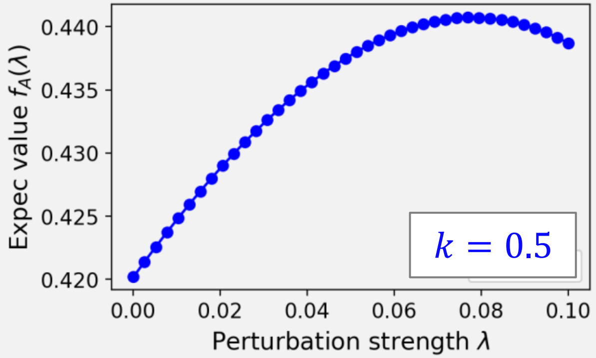
Quantum Kicked Top $\hat{U} = e^{i\alpha \hat{S}_y} e^{i\frac{k}{2S}\hat{S}_z^2}$

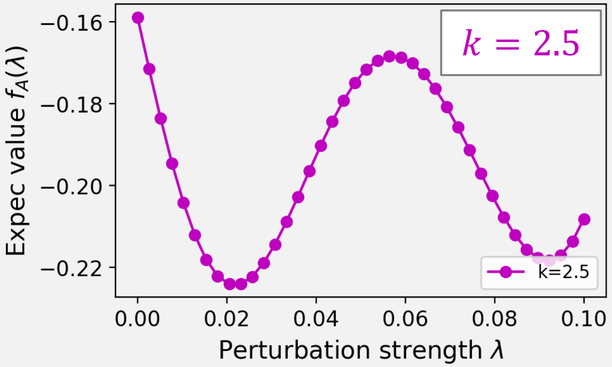


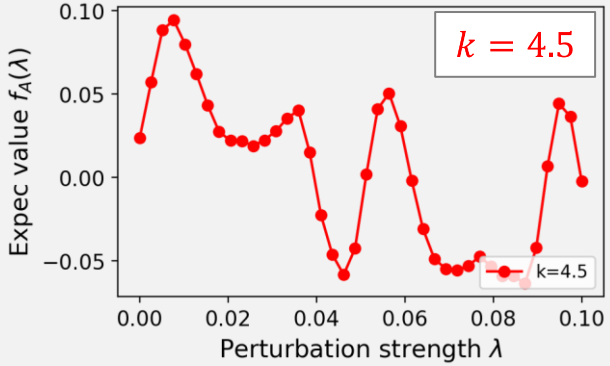
ideal $|\psi_N\rangle = \hat{U}^N |\psi_0\rangle$

perturbed $|\tilde{\psi}_N\rangle = \hat{V}(\lambda)\hat{U} \ldots \hat{V}(\lambda)\hat{U}|\psi_0\rangle$

$\hookrightarrow \hat{V}(\lambda) = e^{i\lambda \hat{S}_z}$

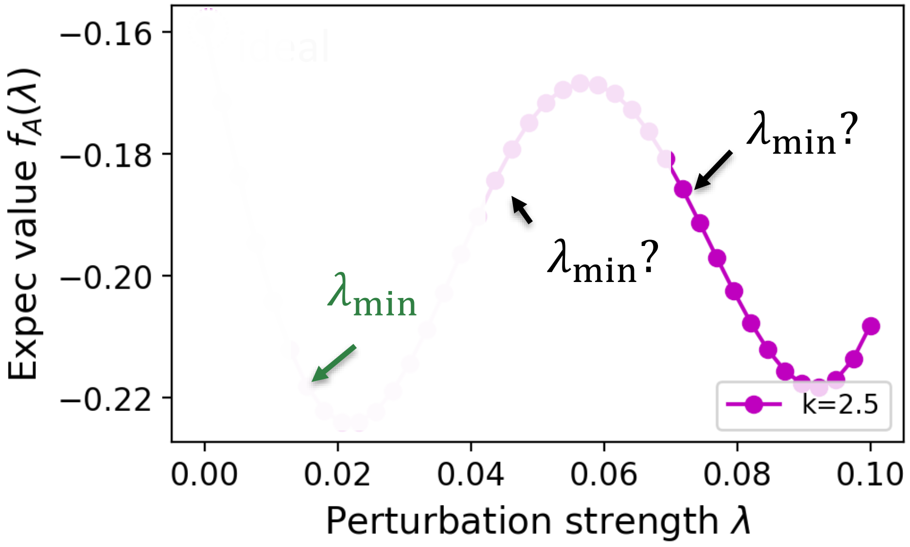




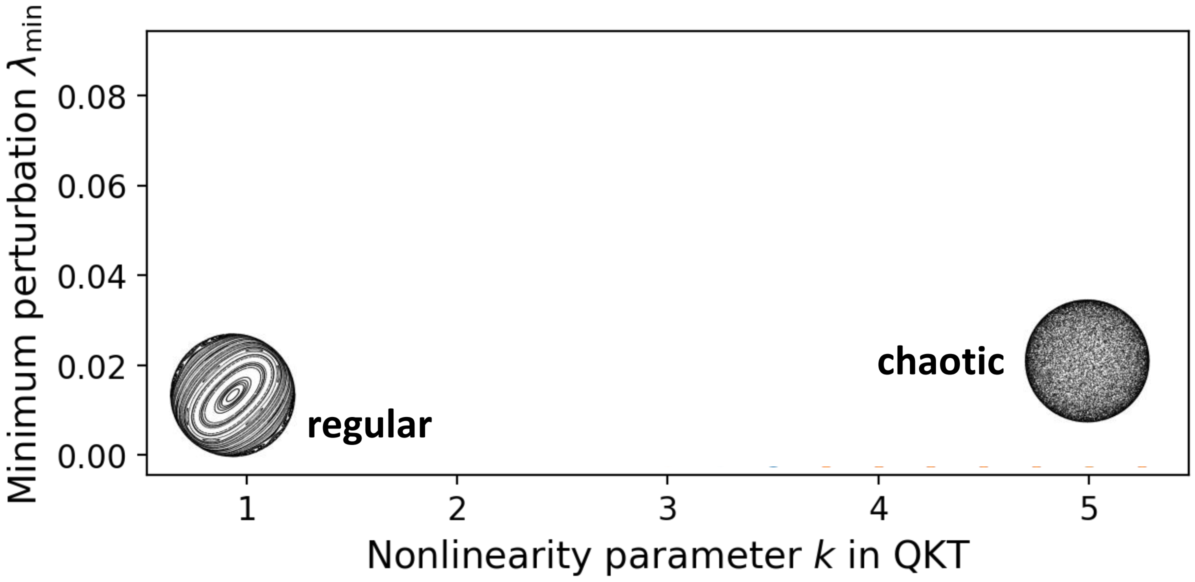


As k increases, $f_A(\lambda)$ curves become more convoluted – one needs to go closer to the ideal case ($\lambda = 0$) to infer the correct result

COMPLEXITY AND ERROR MITIGATION



What is the **minimum value of λ** required to infer the ideal value correctly?
(up to some tolerance)



- ZNE assumes that information about the **noiseless system** can be **decoded** from the **noisy system**
- Here, the method works well only in the **regular** regime
- **Chaotic** systems might be an example on which this **cannot be done** (at least **efficiently** – for instance if $\lambda_{\min}$ decreases with system size)

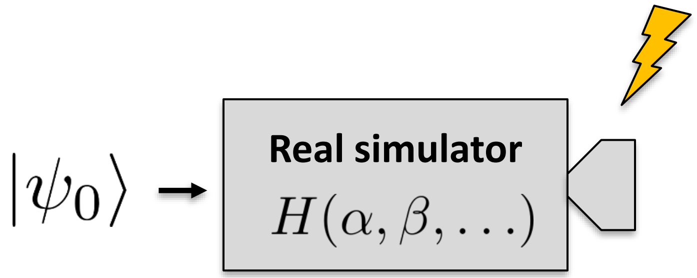
Still lots to explore…

- Different types of errors (coherent / incoherent)
- Connection to perturbation theory breaking down
- Application to many body systems

SUMMARY

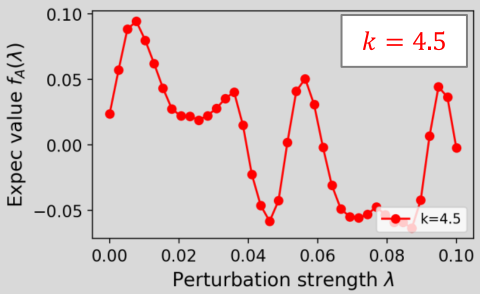
- We developed and implemented a **small, highly accurate quantum simulator** based on optimal control of the internal degrees of freedom of cesium atoms.

- We used this device to explore the **effect of errors and imperfections on the output** of quantum simulators. We developed a theory to predict a priori which **observables might be more robust to errors and which more fragile**. We then went back and tested our predictions in the experiment.



- We are now exploring the interplay between the complexity of the simulated system and the impact of errors in the simulation. In particular, we have been thinking about the role of **chaos.**

Quantum error mitigation



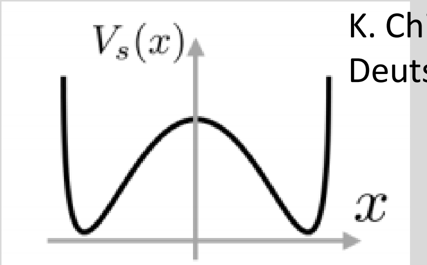
Algorithmic errors, Trotterization and kicked systems

$$e^{i(\alpha S_y+\beta S_z^2)t} \simeq \left(e^{i\alpha H_1\tau} e^{i\beta H_2\tau} \right)^n \quad \text{with } \tau = \frac{t}{n}$$

LMG (integrable) Kicked top (chaotic for large τ)

Heyl, Hauke, Zoller, Sci. Adv. 2019
Sieberer et al, npj Q. Info. 2019

Quantum simulation of phase transitions



K. Chinni, PMP, I. Deutsch 2021

THE UNIVERSITY OF NEW MEXICO

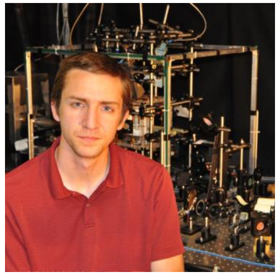
THE UNIVERSITY OF ARIZONA

Karthik Chinni

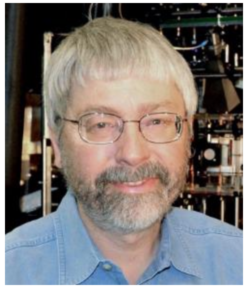
Manuel Muñoz

Ivan Deutsch

Nathan Lysne
(now @ NIST Boulder)



Kevin Kuper



Poul Jessen

THANK YOU FOR YOUR ATTENTION

Some references:

Quantum simulation based on atomic spins: N. Lysne, K. Kuper, PMP, I. Deutsch, P. Jessen, Phys. Rev. Lett. **124** 230501 (2020)

Theory and experiment on robust and fragile observables: PMP, N. Lysne, K. Kuper, I. Deutsch, P. Jessen, PRX Quantum **1**, 020308 (2020)

Chaos quantum simulation of phase transitions: K. Chinni, PMP, I. Deutsch, arxiv: 2103.02714 (2021)

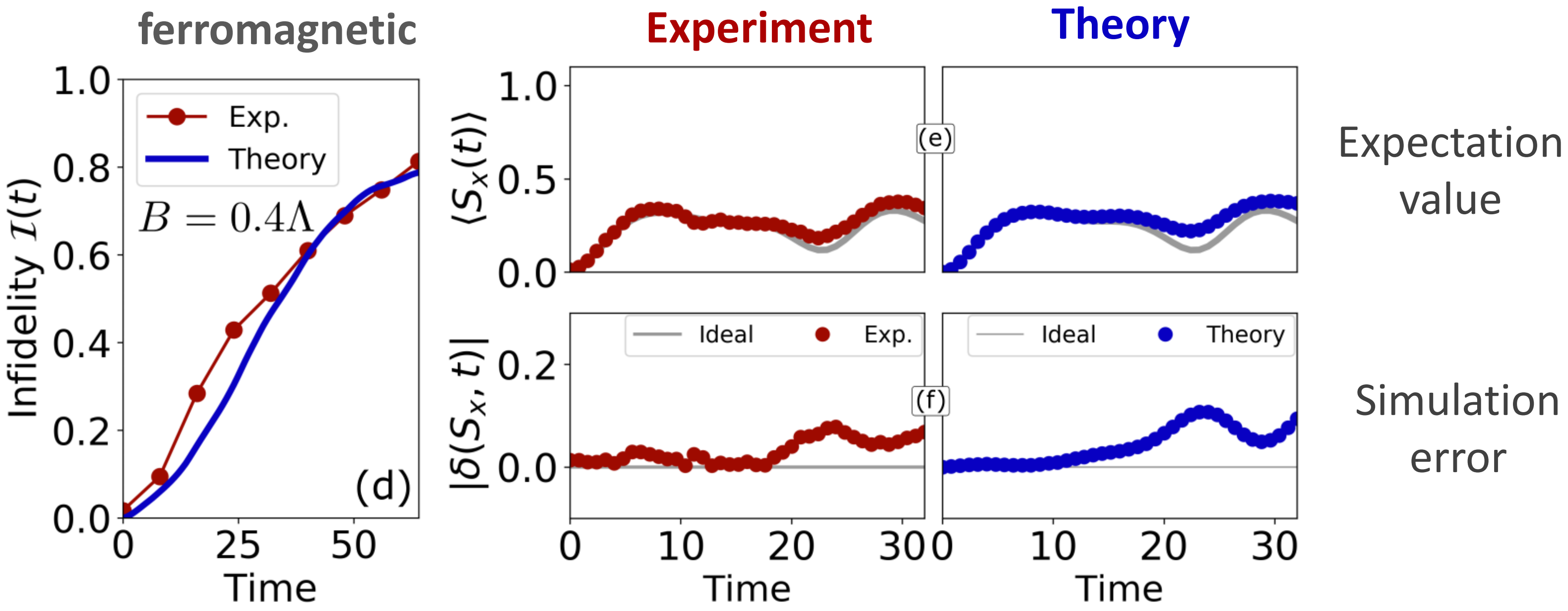
Chaos and Trotterization in collective spin models: M. H. Muñoz-Arias, PMP, I. Deutsch, arxiv: 2103.00748 (2021) – to appear in Phys. Rev. E

EXPERIMENTAL RESULTS

- Quantum simulation of **Lipkin-Meshkov-Glick (LMG)** Hamiltonian

$$H = -\frac{B}{2}\sum_i \sigma_z^{(i)} - \frac{\Lambda}{4N}\sum_{i,j}\sigma_x^{(i)}\sigma_x^{(j)} = -B\,S_z - \frac{\Lambda}{N}S_x^2$$

$$N = 15,\ |\psi_0\rangle = |\downarrow_x\rangle^{\otimes N}$$



Infidelity: $\mathcal{I}(t) = 1 - |\langle\psi(t_k)|\psi_{sim}(t_k)\rangle|^2$

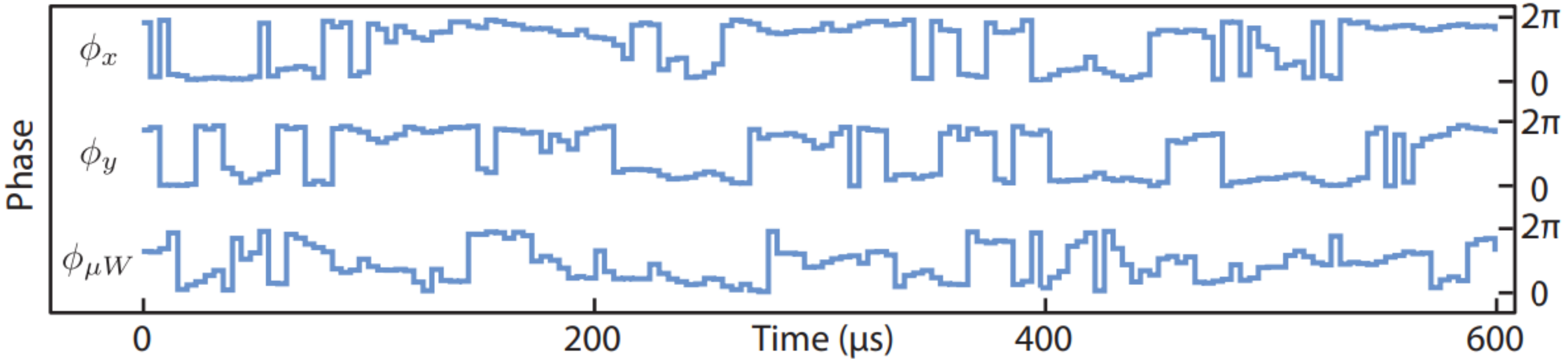
Fitting parameter λ from $\mathcal{I}(t) = 1 - e^{-\lambda^2 t^2}$

- Good agreement with one free parameter
- No info about physical errors used

QUANTUM SIMULATION VIA OPTIMAL CONTROL

Full controllability

Target $W \in \mathrm{SU}(16)$



- Numerical search for control waveforms (**GRAPE**) → maximize $\mathcal{F} = \frac{1}{d^2}\left|\mathrm{Tr}\left(W^\dagger U(T)\right)\right|^2$
- Arbitrarily-chosen 16D random unitaries achieved in $T \sim 600\mu s$ with $\mathcal{F} \sim 0.985$ **(exp)**

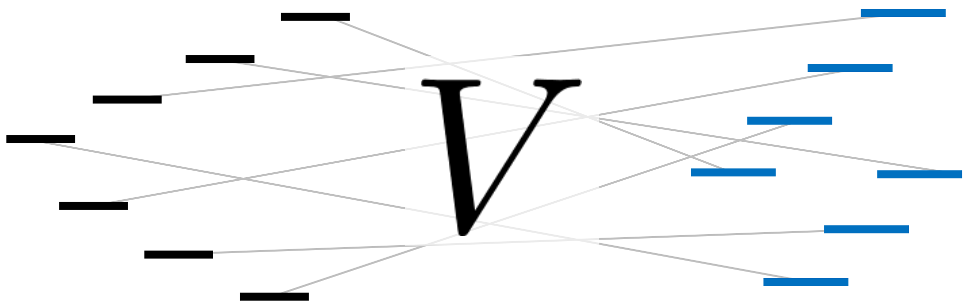
- In particular, we can set $W = e^{-iH_s\Delta t}$
- Simulation → concatenate time steps

$$U_S = W_M \ldots W_2 W_1$$

Simulation Hamiltonian

- Not related to the **physical** (atomic) Hamiltonian
- Any model that 'fits' in a Hilbert space with dim = 16

QUANTUM SIMULATION VIA OPTIMAL CONTROL



For the control optimization, this means that we wish to maximize the overlap between U(T) and

$$W_V = V^\dagger W V$$

where $V \in SU(d)$ sets the **mapping**

We maximize the **fidelity** $\mathcal{F} = \frac{1}{d^2}\left|\mathrm{Tr}\left(W_V^\dagger U(T)\right)\right|^2$ over

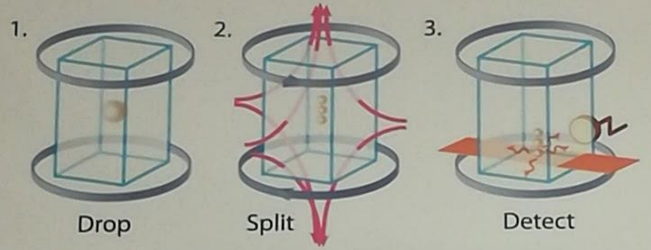
- **Control fields** ⟶ $\{\phi_x(t), \phi_y(t), \phi_{\mu w}(t)\}$
- **Mapping** ⟶ $V = \exp(-iA)$ ⟶ $A = \sum_{i=1}^{d^2-1} \alpha_k \Lambda_k$

Basis of $su(d)$

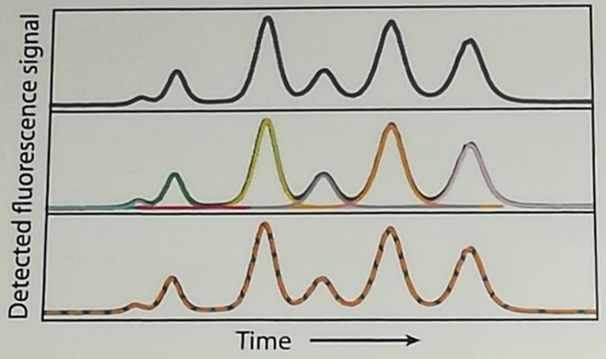
additional $d^2 - 1$ optimization parameters

Measurement

We use Stern-Gerlach Analysis (SGA) on 10^6 identically prepared atoms to estimate the final state populations.



The resulting fluorescence as a function of time is fit to extract the areas under each peak. These areas correspond to the relative populations of each magnetic sublevel.



Preceding SGA with the unitary POVM map yields an estimate of the probabilities

$$p_\alpha = \langle \phi_\alpha | \rho | \phi_\alpha \rangle$$

We use these probabilities to reconstruct a given state. Once reconstructed, we compare the fidelity to the state we intended to make

$$\mathcal{F} = \mathrm{Tr}\left(\sqrt{\sqrt{\rho_e}\rho_{test}\sqrt{\rho_e}} \right)$$